Topology

Back Paper / Supplementary Examination

Instructions: All questions carry ten marks.

1. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is continuous at exactly two points of $\mathbb{R}$. Justify your answer.

2. Define *metrizable* space. Let X be a metrizable space. Prove that there always exist a bounded metric on X which induces the given topology.

3. Let Y be a totally ordered set. Define order topology on Y. Let $f, g : X \to Y$ be two continuous functions with respect to the order topology on Y. Prove that the function $h : X \to Y$, defined by

$$h(x) = \min\{f(x), g(x)\}$$

   is a continuous function.

4. Let $f, g : X \to Y$ be two continuous maps between topological spaces X and Y. Prove that the set $C = \{x \in X \mid f(x) = g(x)\}$ is a closed subset of X if Y is Hausdorff.

5. Define all sepearation axioms (T_1 to T_4). Define first countable and second countable spaces.

6. A map $f : X \to Y$ is called *locally constant* if each point x of X has an open neighbourhood such that the restriction of f to that neighbourhoos is a constant map.

   (a) Show that any locally constant map is continuous.

   (b) Prove that a locally constant map on a connected set is a constant map.

7. Define compact space. Prove that any compact Hausdorff space is normal.

8. Prove that product of two compact spaces is compact.